# Predictability in Unstable, Continuous Systems

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## LONG-TERM GOALS

The long-term goal of my research in this project is to improve our ability to predict environmental conditions using dynamical models.

## OBJECTIVES

The central objective of my research in this project is to understand the mathematical and physical connections between the bred-growing-mode and singular vector techniques recently developed for numerical weather prediction, the Lyapunov vectors and exponents of dynamical systems theory, and instability theories of geophysical fluid dynamics. My intent is to gain insight into fundamental mathematical and physical aspects of predictability in unstable (irregular, chaotic) continuous systems.

## APPROACH

I am using a combination of analytical and numerical methods to study a variety of mathematical models of geophysical fluid flows.

## WORK COMPLETED

I have completed a preliminary study of Floquet and singular vectors in the framework of the periodic orbit analysis of a high-dimensional geophysical fluid model. This model is a two-layer, quasi-geostrophic, numerical channel model with 48 along-channel and 40 cross-channel modes in each layer, for a total of 3,840 degrees of freedom. The model was studied in a strongly nonlinear, unstable regime, in which small disturbances to an unstable, steady, zonal, baroclinic shear flow grow to finite amplitude, equilibrate only statistically, and continue to vacillate irregularly for arbitrarily long times. An approximate unstable periodic orbit (Figure 1, upper panels) was identified in this regime by searching for near returns in a long, chaotic time series. Truncated estimates of the leading Floquet vectors (FVs) of this unstable cycle were computed numerically using a direct approach, by computing the linear evolution over the cycle of an independent set of initial disturbances, and then solving the resulting matrix eigenvalue problem for the disturbance eigenmodes and eigenvalues.

# Report Documentation Page

*Form Approved*
*OMB No. 0704-0188*



| | | |
|---|---|---|
| 1. REPORT DATE<br>**30 SEP 2001** | 2. REPORT TYPE | 3. DATES COVERED<br>**00-00-2001 to 00-00-2001** |

| | |
|---|---|
| 4. TITLE AND SUBTITLE<br>**Predictability in Unstable, Continuous Systems** | 5a. CONTRACT NUMBER<br>5b. GRANT NUMBER<br>5c. PROGRAM ELEMENT NUMBER |
| 6. AUTHOR(S) | 5d. PROJECT NUMBER<br>5e. TASK NUMBER<br>5f. WORK UNIT NUMBER |
| 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)<br>**College of Oceanic and Atmospheric Sciences,,Oregon State University,104 Ocean Admin Bldg,,Corvallis,,OR, 97331** | 8. PERFORMING ORGANIZATION REPORT NUMBER |
| 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) | 10. SPONSOR/MONITOR’S ACRONYM(S)<br>11. SPONSOR/MONITOR’S REPORT NUMBER(S) |

12. DISTRIBUTION/AVAILABILITY STATEMENT
**Approved for public release; distribution unlimited**

13. SUPPLEMENTARY NOTES

14. ABSTRACT

15. SUBJECT TERMS

| 16. SECURITY CLASSIFICATION OF: | | | 17. LIMITATION OF ABSTRACT | 18. NUMBER OF PAGES | 19a. NAME OF RESPONSIBLE PERSON |
|---|---|---|---|---|---|
| a. REPORT<br>**unclassified** | b. ABSTRACT<br>**unclassified** | c. THIS PAGE<br>**unclassified** | **Same as Report (SAR)** | **6** | |

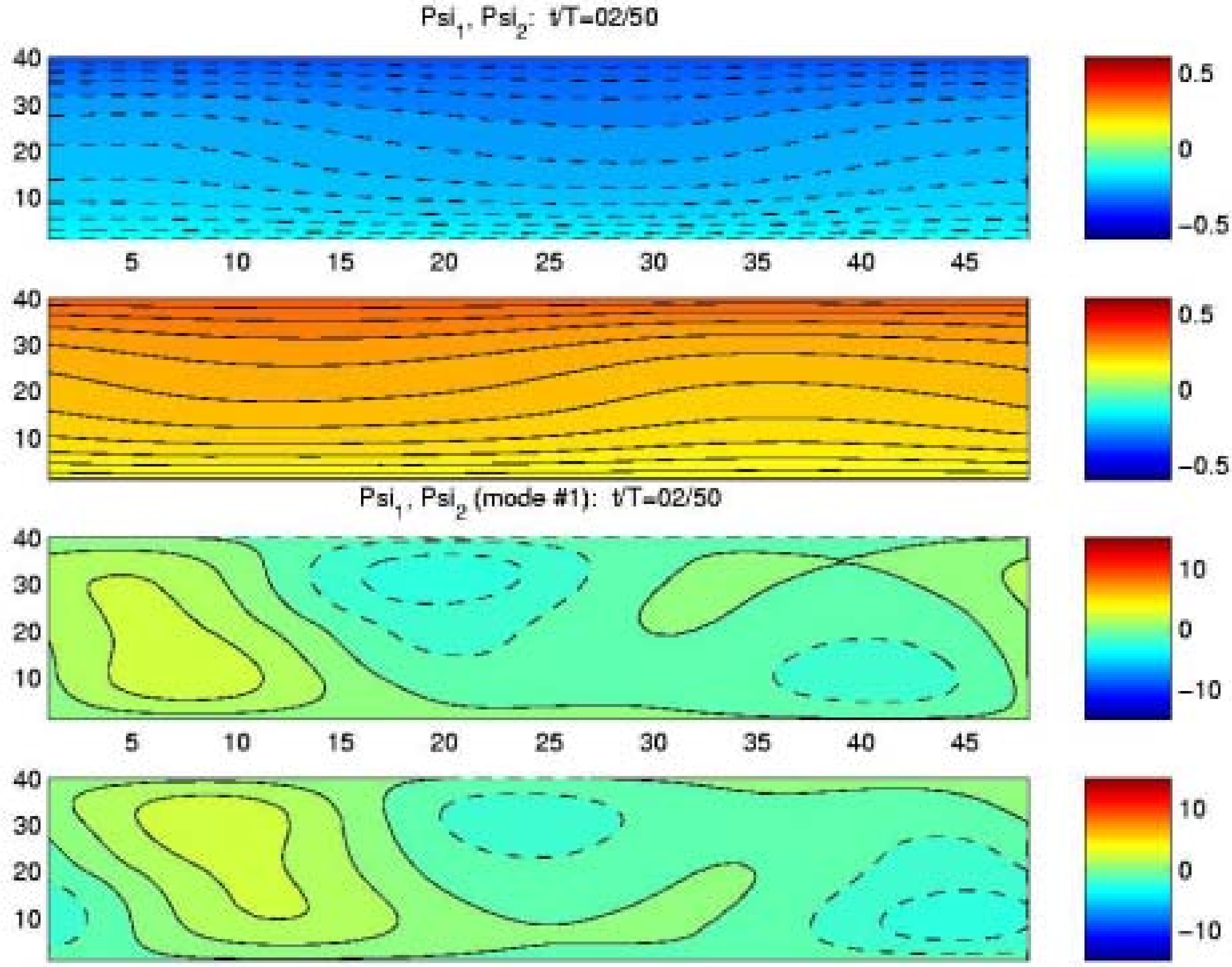


***Figure 1(a). Basic flow streamfunctions (upper two panels) in upper (uppermost panel) and lower layer, and streamfunctions of the time-periodic part of the leading mode of instability FV #1 (lower two panels) in upper and lower (lowermost panel) layers, during onset of baroclinic wave growth.***

Recently, graduate student Christopher Wolfe and I have implemented a multi-dimensional Newton-Picard method adapted from Roose et al. (1995) and Lust et al. (1998) to obtain an improved and more reliable estimate of the unstable cycle. This method uses a Newton iteration in the 12-dimensional subspace spanned by the 12 leading estimated FVs from the above analysis, and a Picard iteration in the 3,828-dimensional, orthogonal, strongly stable subspace. With this approach we have been able to improve the accuracy of the estimate of the periodic cycle by a factor of 500, where the error is measured in terms of the relative norm of the deviation from periodicity, the difference of initial and final states along the cycle.

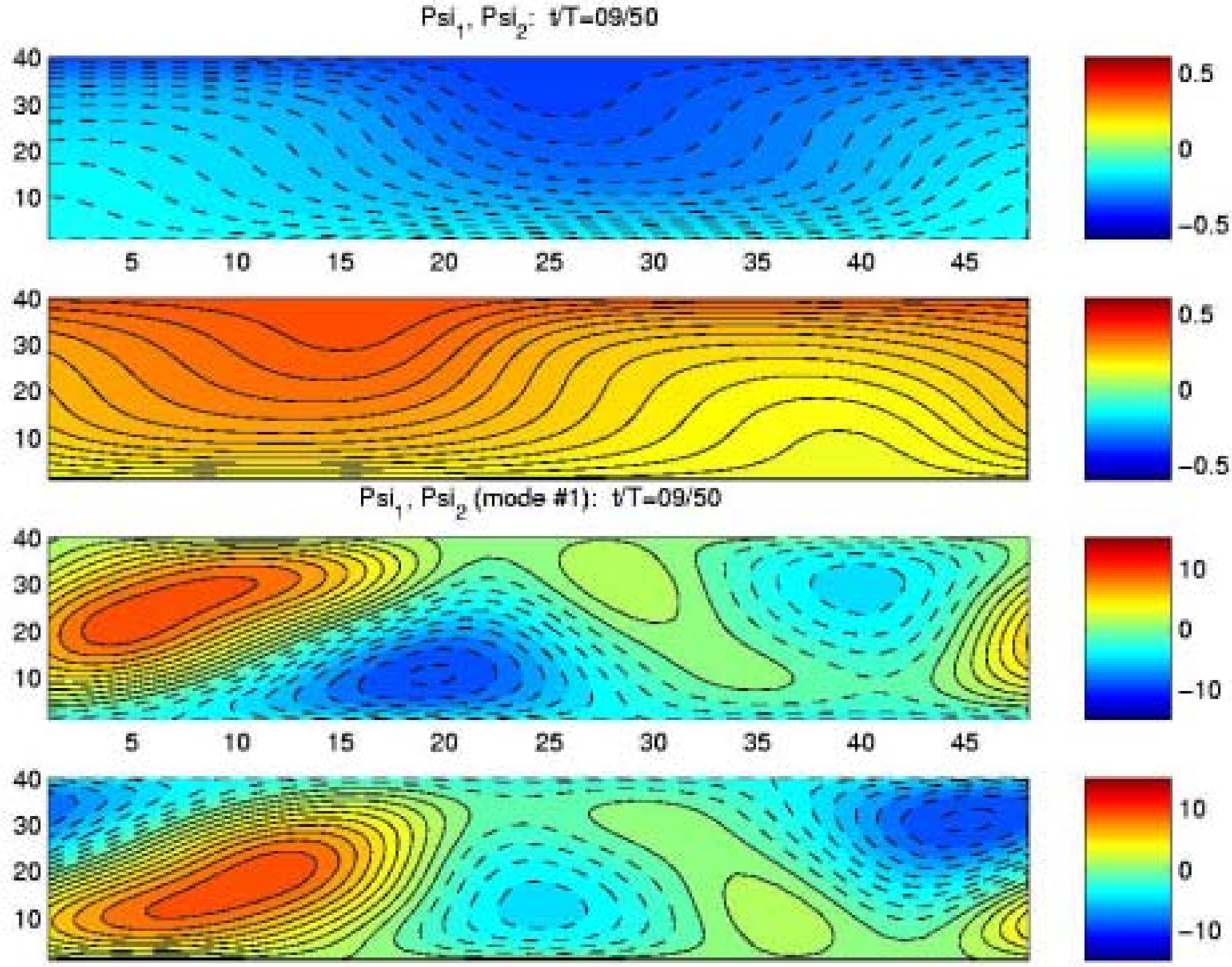


***Figure 1(b). Basic flow streamfunctions (upper two panels) in upper (uppermost panel) and lower layer, and streamfunctions of the time-periodic part of the leading mode of instability FV #1 (lower two panels) in upper and lower (lowermost panel) layers, prior to the baroclinic wave maximum.***

In addition, studies of Lyapunov, Floquet, and singular vectors for weakly nonlinear baroclinic waves, and the collaboration with Eli Tziperman (Weizmann Institute, Israel) on predictability in the Cane-Zebiak coupled tropical ocean-atmosphere model, which were undertaken in earlier years of this project, have been carried through to publication (Samelson, 2001a, 2001b; Samelson and Tziperman, 2001). This project has also provided partial support for several other efforts, including studies of the lower atmosphere and model-scatterometer stress field comparisons along the Oregon coast (http://www-hce.coas.oregonstate.edu/~cmet/index.html; Samelson et al., 2001; see also Bielli et al., 2001), a collaboration with J. S. Allen and graduate student Brandy Kuebel (Oregon State University) on a novel use of dynamical systems techniques to analyze Lagrangian motion in models of coastal ocean circulation, and an incipient collaboration with George Haller (Brown University) on a similar exploration of Lagrangian motion in the unstable, periodic, quasi-geostrophic flow discussed above (Figure 1, upper panels).

## RESULTS

The primary result of this work is the demonstration that, even in the strongly nonlinear case, the analysis of time-dependent, normal-mode disturbances to unstable, time-periodic basic flows is both possible and interesting. This opens a new perspective on the analysis of disturbance growth in time-dependent flows, and on the closely related problem of error growth in predictive models of time-dependent flows. The FVs are time-dependent normal modes of instability for the time-dependent basic flow. The results of this project provide concrete examples of the complex spatio-temporal structure that such time-dependent normal modes can have (Figure 1, lower panels). A comprehensive understanding and physical interpretation of the structure of these FVs and their implications for predictability will require additional analysis.

## IMPACT/APPLICATIONS

The primary potential future impact of these results is on the design and use of ensemble forecasting techniques for the prediction of oceanic and atmospheric conditions.

## TRANSITIONS

George Haller (Brown University) is using dynamical systems techniques to analyze Lagrangian motion in the unstable, periodic solution of the quasi-geostrophic model discussed above (Figure 1, upper panels).

## RELATED PROJECTS

This work is part of the ONR Predictability DRI. The coastal meteorological research (Samelson et al., submitted; http://www-hce.coas.oregonstate.edu/~cmet/index.html) is partially supported by the ONR project "COAMPS Simulations of the Coastal Atmosphere" and the NSF project "COAST: Coastal Ocean Advances in Shelf Transport."

## SUMMARY

The results described above open a new perspective on the analysis of the evolution and predictability of oceanic and atmospheric flows, by showing that techniques previously restricted to highly simplified models can be extended and adapted for models that are sufficiently complex that they can be expected to provide substantial insight into geophysical fluid motion. In the next two years, I hope this perspective will yield specific new results relevant to instability theory and to ensemble forecasting methods for environmental prediction. Participation in this ONR-sponsored work has enhanced my institution by increasing its visibility in the research community, stimulating interaction among institutional colleagues, and attracting a graduate student specifically interested in the problems addressed in this work.

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